GALAXY TUTORIALS

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VECTOR OR CROSS PRODUCT

1. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}, \vec{b}\ \hat{\imath}+2\hat{\jmath}+3\hat{k}$.
. **[ANS : $-\frac{1}{\sqrt{6}}\hat{\imath}+\frac{2}{\sqrt{6}}\hat{\jmath}-\frac{1}{\sqrt{6}}\hat{k}$]**

2. Find a unit vector perpendicular to both the vectors $\hat{\imath}-2\hat{\jmath}+3\hat{k}$ and $\hat{\imath}+2\hat{\jmath}-\hat{k}$. **[ANS : $\frac{1}{\sqrt{3}}(-\hat{\imath}+\hat{\jmath}+\hat{k})$]**

3. Find a unit vector perpendicular to the plane ABC where $A,\ B$ and C are the points $(3,-1,2)$ and $(1,-1,-3), (4,-3,1)$ respectively . **[ANS : $\frac{1}{\sqrt{165}}(-10\hat{\imath}-7\hat{\jmath}+4\hat{k})$]**

4. Find a vector of magnitude 9, which is perpendicular to both the vectors $4\hat{\imath}-\hat{\jmath}+3\hat{k}$ and $-2\hat{\imath}+\hat{\jmath}-2\hat{k}$ . .
. **[ANS :$-3\hat{\imath}+6\hat{\jmath}+6\hat{k}$.]**

5. Let $\vec{a}=\hat{\imath}-\hat{\jmath}, \vec{b}=3\hat{\jmath}-\hat{k}$ and $\vec{c}=7\hat{\imath}-\hat{k}$. find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=1$. .
. **[ANS : $\vec{d}=\frac{1}{4}(\hat{\imath}+\hat{\jmath}+3\hat{k})$.]**

6. Find the area of the parallelogram determined by the vectors $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ and $3\hat{\imath}-2\hat{\jmath}+\hat{k}$. .
. **[ANS : $8\sqrt{3}$ square units .]**

7. If $A(0,1,1)$ $B(2\ ,3\ ,-2)$ $C\ (22\ ,19,-5)$ and $D(1,-2,1)$ are the vertices of a quadrilateral$ABCD$, find its area . . .
. **[ANS : $\sqrt{3160}$ square units .]**

8. Show that $(\vec{a}\times\vec{b})^2=|\vec{a}|^2|\vec{b}|^2-(\vec{a}.\vec{b})^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b}\\ \vec{a}.\vec{b} & \vec{b}.\vec{b}\end{vmatrix}$.

9. If $\vec{a}\ ,\vec{b},\vec{c}$ are the position vectors of the vertices A, B, C of a triangle ABC, show that the area of triangle ABC is $\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$. Deduce the condition for points $\vec{a}\ ,\vec{b}\ ,\vec{c}$ to be collinear

10. Show that distance of the point $\vec{c}$ from the line joining $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}{|\vec{b}-\vec{a}|}$

11. Prove that the points A, B and C with position vectors $\vec{a}\ ,\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$.

12. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$

13. If $\vec{a}\ ,\vec{b}\ ,\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

14. If $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}, \vec{a}\neq\vec{0}$ and $\vec{b}\neq\vec{c}$, show that $\vec{b}=\vec{c}+t\vec{a}$ for some scalar t

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1. In a triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{C}{\sin C}$
2. Prove by vector method that the parallelogram on the same base and between the same parallels are equal in area .
3. If D , E, F are the mid –point of the sides BC, CA and CA and AB respectively of a triangle ABC, prove by vector method that Area of $\Delta DEF = \frac{1}{4}$(Area of ΔABC).
4. Using vectors : prove that if a , b , c are the lengths of three sides of a triangle , then its area Δ is given by $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $2s = a + b + c$.
5. (i) Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$. **[ANS : $\frac{5}{3}(32\hat{\imath} - \hat{\jmath} + 14\hat{k})$]**

   (ii) Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a} = 21$. **[ANS : $-\frac{1}{3}(\hat{\imath} - 16\hat{\jmath} - 13\hat{k})$]**