

INCREASING AND DECREASING FUNCTIONS

1. Find the intervals in which $f(x) = (x + 1)^3(x - 3)^3$ is increasing or decreasing.
 [Ans : $f(x)$ is increasing on $(1, 3) \cup (3, \infty)$ $f(x)$ is decreasing on $(-\infty, -1) \cup (-1, 1)$]
2. Find the intervals in which the function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing or decreasing.
 [Ans: $f(x)$ is increasing on $(-1, 0) \cup (0, \infty)$]
3. Separate $[0, \pi/2]$ into subintervals in which $f(x) = \sin 3x$ is increasing or decreasing.
 [Ans: $f(x)$ is increasing on $(0, \pi/6)$, $f(x)$ is decreasing on $(\pi/6, \pi/2)$]
4. Find the intervals in which the function f given by $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$, $0 \leq x \leq 2\pi$ is
 (i) increasing (ii) decreasing
 [Ans: $f(x)$ is increasing on $(0, \pi/2) \cup (3\pi/2, 2\pi)$, $f(x)$ is decreasing on $(\pi/2, 3\pi/2)$]
5. Separate the interval $[0, \pi/2]$ into sub - intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.
 [Ans : $f(x)$ is decreasing on $[0, \pi/4]$. $f(x)$ is increasing on $[\pi/4, \pi/2]$]
6. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is increasing or decreasing.
 [Ans : $f(x)$ is increasing on $(0, \pi/4) \cup (5\pi/4, 2\pi)$, $f(x)$ is decreasing on $(\pi/4, 5\pi/4)$]
7. Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or decreasing.
 [Ans : $f(x)$ is strictly increasing on $(0, \frac{\pi}{4}) \cup (\frac{7\pi}{12}, \frac{11\pi}{12})$ $f(x)$ is strictly decreasing on $(\frac{\pi}{4}, \frac{7\pi}{12}) \cup (\frac{11\pi}{12}, \pi)$]
8. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on the interval $(0, \pi/4)$.
 [Ans : $f(x)$ is increasing on $(0, \pi/4)$]
9. Prove that $f(\theta) = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$ is an increasing function of θ in $[0, \frac{\pi}{2}]$
 [Ans : $f(\theta)$ is increasing on $[0, \pi/2]$].
10. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $(-\pi/3, \pi/3)$
 [Ans: f is strictly decreasing on $(-\pi/3, \pi/3)$].
11. Find the values 'a' for which the function $f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ decrease for all real values of x .
 [Ans: $f(x)$ decreases for all $x \in R$, if $a \in (-\infty, -3)$]
12. Find the values of k for which $f(x) = kx^3 - 9kx^2 + 9x + 3$ is increasing on R .
 [Ans : $f(x)$ is increasing on R , if $k \in (0, 1/3)$]