'PEACE'
HOMERTON GRAMMAR SCHOOL
PERIODIC TEST II (2021-22)
CLASS: XII, SUBJECT: MATHEMATICS

TIME: 1 Hr. 20 min **M.M :-50**

NAME:_____ **ROLL No._____**

Question 1.The function $f : A \to B$ defined by $f(x) = 4x + 7$, $x \in R$ is
(a) one-one (b) Many-one (c) Odd (d) Even

Question 2.The smallest integer function $f(x) = [x]$ is
(a) One-one (b) Many-one (c) Both (a) & (b) (d) None of these

Question 3.The function $f : R \to R$ defined by $f(x) = 3 - 4x$ is
(a) Onto (b) Not onto (c) None one-one (d) None of these

Question 4.The number of bijective functions from set A to itself when A contains 106 elements is
(a) 106 (b) $(106)^2$ (c) 106! (d) 2^{106}

Question 5.Let us define a relation R in R as a R b if $a \geq b$. Then R is
(a) an equivalence relation
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) neither transitive nor reflexive but symmetric

Question 6.Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}. Then R is
(a) reflexive but not symmetric (b) reflexive but not transitive
(c) symmetric and transitive (d) neither symmetric, nor transitive

Question 7. If $\cos^{-1} x + \sin^{-1} x = \pi$, then the value of x is
(a) 3/2 (b) $1\sqrt{2}$ (c) $\sqrt{3}/2$ (d) $2/\sqrt{3}$

Question 8.If $\sin^{-1} x - \cos^{-1} x = \pi/6$, then x =
(a) ½ (b) $\sqrt{3}/2$ (c) $-1/2$ (d) $-\sqrt{3}/2$

Question 9.If $\tan^{-1}(\cot \theta) = 2\theta$, then θ is equal to
(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) None of these

Question 10. $\text{Cot}(\pi/4 - 2\cot^{-1} 3) =$
(a) 7 (b) 6 (c) 5 (d) None of these

Question 11. If $\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$, then x =
(a) 5 (b) 1/5 (c) 5/14 (d) 14/5

Question 12.

$\sin^{-1}\left(\frac{-1}{2}\right)$

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

Question 13.

$\cos^{-1}\left(\frac{1}{2}\right)$

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Question 14.

$\tan^{-1}(\sqrt{3})$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

Question 15.

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Question 16.

$\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 6π

Question 17.

$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Question 18.

The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

(a) 1 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Question 19.

The derivative of

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) \text{ with respect to } \tan^{-1}\left(\frac{2x}{1-x^2}\right) \text{ is}$$

(a) 0 (b) 1

(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

Question 20.

If $x = a\sin\theta$ and $y = b\cos\theta$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{a}{b^2}\sec^2\theta$ (b) $\frac{b}{a}\sec^2\theta$

(c) $\frac{b}{a^2}\sec^3\theta$ (d) $-\frac{b}{a^2}\sec^3\theta$

Question 21.

If $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{\pi}{4}$

(c) 0 (d) 1

Question 22.

If $y = e^{3x+7}$, then the value of $\left.\frac{dy}{dx}\right|_{x=0}$ is

(a) 1 (b) 0

(c) -1 (d) $3e^7$

Question 23.

If $f(x) = \frac{\sqrt{4+x}-2}{x}$, $x \neq 0$ be continuous at x = 0, then f(0) =

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 2 (d) $\frac{3}{2}$

Question 24.

If $y = e^{x+e^{x+e^{x+\ldots \text{ to } \infty}}}$, find $\frac{dy}{dx}$ =

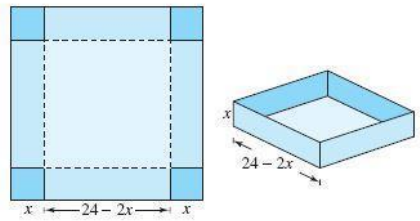
(a) $\frac{y^2}{1-y}$ (b) $\frac{y^2}{y-1}$

(c) $\frac{y}{1-y}$ (d) $\frac{-y}{1-y}$

Answer questions 25-29 based on the following case study:

A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form

a box.



Question 25.Volume of open box formed by folding up the flap:

a. $4(x^3 – 24x^2 + 144x)$ b. $4(x^3 – 34x^2 + 244x)$

c. $x^3 – 24x^2 + 144x$ d. $4x^3 – 24x^2 + 144x$

Question26. In the first derivative test, if dy/dx changes its sign from positive to negative as x increases through c_1, then function attains a:

a .Local maxima at $x = c_1$ b .Local minima at $x = c_1$

C. Neither maxima nor minima at $x = c_1$ d. None of these

Question 27. What should be the side of the square piece to be cut from each corner of the board to behold the maximum volume?

a)14 cm b) 12 cm c) 4 cm d) 5 cm

Question 28. What should be the maximum volume of open box?

a)1034 cm^3 b)1024 cm^3 c) 1204 cm^3 d) 4021 cm^3

Question 29. The smallest value of the polynomial $x^3 – 18x^2 + 96x$ in [0, 9] is:

a)126 b. 0 c .135 d. 160

Question 30.The curve $y – x^{1/5}$ at (0, 0) has

(a) a vertical tangent (parallel to y-axis)
(b) a horizontal tangent (parallel to x-axis)
(c) an oblique tangent
(d) no tangent

Question 31.The equation of normal to the curve $3x^2 – y^2 = 8$ which is parallel to the line , $x + 3y = 8$ is

(a) $3x – y = 8$ (b) $3x + y + 8 = 0$ (c) $x + 3y \pm 8 = 0$ (d) $x + 3y =$

Question 32. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1) then the value of a is

(a) 1 (b) 0 (c) -6 (d) 6

Question 33. The absolute maximum value of $y = x^3 – 3x + 2$ in $0 \le x \le 2$ is

(a) 4 (b) 6 (c) 2 (d) 0

Question 34. The line $y = x + 1$ is a tangent to the curve $y2 = 4x$ at the point

(a) (-1, 2) (b) (1, 2) (c) (1, -2) (d) (2, 1)

Question 35. The function $f(x) = 4x + 3$, $x \in R$ is an increasing function.

(a) true (b) false. (c) none of these

Question 36. The function $f(x) = \log(\cos x)$ is increasing function for $[0, \pi/2]$

(a) true (b) false. (c) none of these

Question 37. Find the maximum and minimum value of the function $y = |x – 3| + 7$, $x \in R$.

(a) (7 , 3) (b) (7 , no minimum value.)

(c) (no maximum value ,7) (d) none of these

Question 38 The function $f(x) = x^2$, for all real x, is

(a)Neither decreasing nor increasing (b) Increasing

(c) Decreasing (d) None of these

Question 39. The slope of the tangent to the curve $x = a\sin t$, $y = a\{\cos t + \log(\tan t/2)\}$ at the point 't' is

a) tan t/2 b) none of these c)tan t d) cot t

Question 40 The function $f(x) = x^2 - 2x$ is strict decreasing in the interval

a) none of these b) R c) $[1,\infty)$ d) $(-\infty, 1)$

Answer questions 41- 45 based on the following case study:

A relation R on a set A is said to be an equivalence relation on A if it is

- Reflexive i.e , (a,a) belongs to R for all a belongs to A .
- Symmetric i.e (a,b) belongs to R it implies (b,a) belogs to R For all (a,b) belongs to A.
- Transitive i.e (a,b) belongs to R and (b,c) belongs to R it implies (a,c) belongs to R For a,b,c belongs to A.

Based on the above information answer the following questions

(i) If the relation R = { (1,1) , (1,2) , (1,3) , (2,2) , (2,3), (3,1), (3,2), (3,3)} defined on the set A = { 1,2,3} ,then R is

[a] reflexive [b] symmetric [c] transitive [d] equivalence

(ii) If the relation R = { (1,2) (2,1) ((1,3), (3,1)} defined on the set A = { 1,2,3} ,then R is

[a] reflexive [b] symmetric [c] transitive [d] equivalence

(iii) If the relation R on the set N of all natural numbers defined as R = {(x,y) : y=x+5 and x<4} then R is

[a] reflexive [b] symmetric [c] transitive [d] equivalence

(iv) If the relation R on the set A = {1,2,3….,13,14} defined as R={(x,y):3x –y =0 } then R is

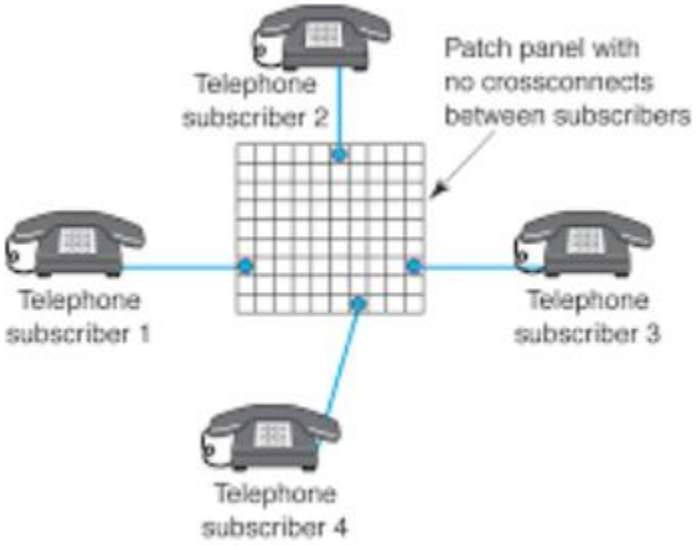
[a] reflexive [b] symmetric [c] transitive [d] none of these

(v) If the relation R on the set A ={1,2,3} defined as R ={(1,1),(,2),(1,3),(2,1),(2,2),(2,3), (3,1), (3,2),(3,3)} then R is

[a] reflexive only [b] symmetric only [c] transitive only [d] equivalence

Answer questions 46-50 based on the following case study:

A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increases the annual subscription and it is believed that for every increase of 1 one subscriber will discontinue the service.



i. If x be the annual subscription then the total revenue of the company after increment will be:
   a. $R(x) = -x^2 + 200x + 150000$ c.$R(x) = x^2 - 200x - 140000$
   b. $R(x) = 200x^2 + x + 150000$ d.$R(x) = -x^2 + 100x + 100000$

ii. To find maximum profit we put

   a. $R'(x) = 0$ b.$R'(x) > 0$ c.$R'(x) < 0$ d.$R''(x) = 0$

iii. How much fee the company should increase to have maximum profit?

   a. Rs. 150 b.Rs. 100 c.Rs. 200 d.Rs. 250

iv. Find the maximum profit that the company can make if the profit function is given by $P(x) = 41 + 24x - 18x^2$.

   a. 25 b.44 c.45 d.49

v. Find both the maximum and minimum value respectively of $3x^4 - 8x^3 + 48x + 1$ on the interval [1, 4].

   a. -63, 257 b.258, -63 c.257, -63 d.-63, -257