VIDYA MANDIR PUBLIC SCHOOL, SECTOR 15/A, FARIDABAD

CLASS XII [2021-22] FIRST TERM EXAMINATION ROLL NO.....3

TIME FOR OBJECTIVE PAPER: 90 MIN. SUBJECT – MATHEMATICS SET A M.M.40

General Instructions: i) All questions are compulsory.
ii) All questions carry 1 mark.
iii) There is no negative marking.

1. Let R be the relation is the set {1, 2 ,3 ,4 } given by R = {(1,2),(2,2), (1,1),(4,4),(1 ,3), (3 , 3),(3,2)}.

   Choose the correct answer.

   a. R is reflexive and symmetric but not transitive.

   b. R is reflexive and transitive but not symmetric.

   c. R is transitive and symmetric but not reflexive.

   d. R is an equivalence relation.

2. Let $P=\{(x,y): x^2+y^2=1,\ x,y\in R\}$ .Then P is

   a. Reflexive b. symmetric c. transitive d. anti – symmetric

3. The relation R defined in the set A = { 1,2,3,4,5,6,7} by

   R = {(a , b) : both a and b are either odd or even}.Then R is :,

   a. Symmetric b. transitive c. an equivalence relation d. reflexive

4. A function f from the set of natural numbers to integers is defined by

$$f(n)=\begin{cases}\frac{n-1}{2} & \text{when n is odd}\\ -\frac{n}{2} & \text{when n is even}\end{cases}$$ is

   a. One-one but not on to
   b. on to but not one-one
   c. one-one and on to both
   d. neither one-one nor onto

5. If $A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ and $A^2-KA-5I=0$, *then* K =

   a. 5 b. 3 c. 7 d. none of these

6. If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is such that $A^2=I$, then :

   a. $1+\alpha^2+\beta\gamma=0$ b. $1-\alpha^2+\beta\gamma=0$ c. $1-\alpha^2-\beta\gamma=0$ d. $1+\alpha^2-\beta\gamma=0$

7. If $U=\begin{bmatrix}2 & -3 & 4\end{bmatrix}, V=\begin{bmatrix}3\\2\\1\end{bmatrix}, X=\begin{bmatrix}0 & 2 & 3\end{bmatrix}$ and $Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$, then the value of UV + XY is :

   a. 20 b. [-20] c. -20 d. [20]

8. If $2A+3B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}$ and $A+2B=\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix}$, then B is

   a. $\begin{bmatrix}8 & -1 & 2\\ -1 & 10 & -1\end{bmatrix}$ b. $\begin{bmatrix}8 & 1 & 2\\ -1 & 10 & -1\end{bmatrix}$ c. $\begin{bmatrix}8 & 1 & -2\\ -1 & 10 & -1\end{bmatrix}$ d. $\begin{bmatrix}8 & 1 & 2\\ 1 & 10 & 1\end{bmatrix}$

9. $\cos\theta\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & \cos\theta\\ -\cos\theta & \sin\theta\end{bmatrix}$ is equal to

a. $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ b. $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ c. $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ d. none of these

10. For what value of K is the function $f(x)=\begin{cases}\frac{\sin 5x}{3x}+\cos x & x\neq 0\\ k & x=0\end{cases}$ continuous at x=0?

a. $\frac{3}{8}$ b. $\frac{5}{8}$ c. 1 d. $\frac{8}{3}$

11. If $xy^2=ax^2+bxy+y^2$, then find $\frac{dy}{dx}$

a. $\frac{2ax+by+y^2}{2xy+bx+2y}$ b. $\frac{2ax+by-y^2}{2xy-bx-2y}$ c. $\frac{ax+by+xy}{xy+x^2+y^2}$ d. $\frac{2x^2+axy+y^2}{x^2+y^2+2xy}$

12. Derivative of $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]$ with respect to x is

a. 0 b. 1/2 c. -1/2 d. -1

13. $\frac{d}{dx}\left(\tan^{-1}\left(\frac{\sqrt{x}-\sqrt{a}}{1+\sqrt{xa}}\right)\right)$ Where $x,a>0$, is equal to

a. $\tan^{-1}\sqrt{x}+\tan^{-1}\sqrt{a}$ b. $\frac{1}{1+x}$ c. $\frac{1}{1+x}+\frac{1}{1+a}$ d. $\frac{1}{2\sqrt{x}(1+x)}$

14.If $x^x=y^y$, then $\frac{dy}{dx}$ is equal to

a. $-\frac{y}{x}$ b. $-\frac{x}{y}$ c. $1+\log\left(\frac{x}{y}\right)$ d. $\frac{1+\log x}{1+\log y}$

15. A corner point of a feasible region is a point in the region is which is the_____ of two boundary lines.

a. union b. difference c. intersection d. none of these

16. The corners points of the feasible region determined by the system of linear constraints are (0,10), (5,5), (15,15), (0,20). Let Z = px + qy, where p, q > 0 . condition on p and q so that the maximum of Z occurs at both the points (15,15) and (0,20) is

a. p = q b. p = 2q c. q = 2p d. q = 3p

17. The function f: R -----→R defined by f(x) = 3- 4x is

a. on to b. not on to c. not one-one d. none of these

18. $\sin^{-1}\left(-\frac{1}{2}\right)=$

a. $\frac{\pi}{3}$ b. - $\frac{\pi}{3}$ c. $-\frac{\pi}{6}$ d. $-\frac{5\pi}{6}$

19. $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$ is equal to

a. π b. $-\frac{\pi}{3}$ c. $\frac{\pi}{3}$ d. $\frac{2\pi}{3}$

20. If $x^y=e^{x-$ $1+$

20. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

a. $\frac{1+x}{1+\log x}$ b. $\frac{1-\log x}{1+\log x}$ c. not defined d. $\frac{\log x}{(1+\log x)^2}$

21. If $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$, then $\frac{d^2y}{dx^2}$

a. $\frac{\sec^3\theta}{a\theta}$ b. $\frac{\sec^3\theta}{\theta}$ c. $a\theta\cos^3\theta$ d. $\frac{\sec^3\theta}{a}$

22. Find the point on the curve $y = x^2 + 1$ at the tangent drawn makes 45° angle from x-axis.

a. (1,1/2) b. (1/2,5/4) c. (5/6,3/4) d. (1/4,1/2)

23. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is

a. $[-1,\infty)$ b. $(-2,-1)$ c. $(-\infty,-2]$ d. $[-1,1]$

24. Find the maximum and minimum value of f(x) = x + sin2x in the interval $[0,2\pi]$.

a. max. value = 2π, min. value = 0 b. max. value =0, min. value = 2π

c. max. value =0, min. value = 0 d. none of these

25. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A+A' = I_2$, if the value of α is

a. $\frac{\pi}{6}$ b. $\frac{\pi}{3}$ c. π d. $\frac{3\pi}{2}$

26. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ is the sum of a symmetric matrix B and a skew -symmetric matrix c, then c is

a. $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 1 & -3/2 \\ 5/2 & 1 \end{bmatrix}$

27. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew – symmetric, then the value of a and b are

a. -2 and 3 b. 2 and -3 c. -3 and 4 d. $\frac{1}{2}$ and -3

28. If A_{ij} denotes the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then value of

$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ is

a. 0 b. 5 c. 10 d. -5

29. Tangents to the curve $x^2 + y^2 = 2$ at the points (1,1) and (-1,1) are

a. parallel b. perpendicular c. intersecting but not at right angles d. none of these

30. The interval in which $y = x^2e^{-x}$ is increasing is

a. $(-\infty,\infty)$ b. $(-2,0)$ c. $(2,\infty)$ d. $(0,2)$

31. A vertex of a feasible region by the linear constraints $3x+4y \le 18, 2x+3y \ge 3$ and $x, y \ge 0$ is

a. (0,2) b. (4.8,0) c. (0,3) d. none of these

32. The shaded region for the inequality $x+5y \le 6$ is

a. to the non origin side of $x + 5y = 6$

b. to the either side of $x + 5y = 6$

c. to the origin side of $x + 5y = 6$

d. to the neither side of $x + 5y = 6$

33. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

a. $\frac{7\pi}{6}$ b. $\frac{5\pi}{6}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{6}$

34. Find x, if $\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ is singular

a. 1 b. 2 c. 3 d. 4

35. Region represented by $x \ge 0, y \ge 0$

a. first quadrant b. second quadrant c. third quadrant d. fourth quadrant

CASE STUDY BASED QUESTION

A carpenter designs a window in the form of a rectangle surmounted by a semicircle. The total perimeter of the window is 10m.

36. The perimeter of window in terms of x and y is

a. $2x+2y+\frac{\pi x}{2}$ b. $x+2y+\frac{\pi x}{2}$ c. $2x+y+\frac{\pi x}{2}$ d. $x+2y+\pi x$

37. The value of y in terms of π and x

a. $10-\left(\frac{\pi+2}{4}\right)x$ b. $5-\left(\frac{\pi+2}{2}\right)x$ c. $5-\left(\frac{\pi+2}{4}\right)x$ d. none of these

38. Area of the window through which light enters is

a. $xy+\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ b. $xy+\pi\left(\frac{x}{2}\right)^2$ c. $xy+\frac{1}{2}\pi x^2$ d. $2xy+\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

39. For maximum light x should be

a. $\frac{10}{\pi+4}$ b. $\frac{20}{\pi+4}$ c. $\frac{10}{\pi+2}$ d. $\frac{20}{\pi+2}$

40. For maximum light, the height of the window is

a. $\frac{10}{\pi+4}$ b. $\frac{20}{\pi+4}$ c. $\frac{30}{\pi+4}$ d. none of these

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TIME FOR SUBECTIVE PAPER: 90 MIN SUBJECT – MATHEMATICS SET A M.M.40

General Instructions: i) All questions are compulsory.

ii) There are three sections and marks are allotted against each section.

SECTION – A [2 MARKS QUESTIONS]

1. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

2. Find x, $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=0$

3. Find the intervals in which the function $f(x)=2x^3-9x^2+12x+15$ are strictly increasing or decreasing

4. If $x=\sqrt{a^{\sin^{-1}t}}$, $y=\sqrt{a^{\cos^{-1}t}}$, a > 0 and -1 < a <1, then find $\frac{dy}{dx}$

SECTION – B [3 MARKS QUESTIONS]

5. Show that the relation S in the set $A=\{x\in z:0\le x\le 12\}$ given by $S=\{(a,b):a,b\in A,|a-b| \text{ is divisible by } 4\}$ is an equivalence relation.

6. If $x^{16}y^9=(x^2+y)^{17}$, prove that $\frac{dy}{dx}=\frac{2y}{x}$

7. If $A=\begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C=\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, find AC, BC and (A+B)C

8. let A = R - {3} and B= R – {1}. Consider the function f:A----→B defined by $f(x)=\frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

SECTION – C [5 MARKS QUESTIONS]

9. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.
   (i) what number of rackets and bats must be made if the factory is to work a full capacity?
   (ii) If the profit on a racket and on a bat is Rs 20. and Rs.10 respectively, find the maximum profit of the factory when it works at full capacity.

10. Solve the system by matrix method
    3x – 2y + 3z = 8, 2x + y – z = 1, 4x – 3y + 2z = 4

11. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

12. (i) Prove that the curves $x=y^2$ and xy=k cut at right angles if $8k^2=1$
    (ii) Find the equation of tangent to the curve $y=x^2-2x+7$ which is perpendicular to the line
    5y – 15x = 13