Delhi Public School
FARIDABAD

STD: XII | SUBJECT: Mathematics | ASSESSMENT: Semester 1 optional

Maximum marks : 80 | 20/9/2021 8:05 - 20/9/2021 10:35 | Time Limit : 150 Minutes

General Instructions:

This question paper contains two Parts A and B. Each part is compulsory. Part A carries 56 marks and Part B carries 24 marks.

2. Part A has Objective Type Questions and Part B has Descriptive Type Questions.

Part A :

1. It consists of two sections - I and II.
2. Section I comprises of 20 objective type questions.
3. Section II contains 2 case studies. Each case study comprises of 4 case-based MCQs.

Part B :

1. It consists of two sections - III, and IV.
2. Section III comprises of 3 questions of 3 marks each.
3. Section IV comprises of 3 questions of 5 marks each.
4. Internal choice is provided in 1 question of Sections - III, and IV. You have to attempt only one of the alternatives.

Part A

56 Marks

Section I

40 Marks

Questions in this section carry 2 marks each.

1 For real numbers a and b, define aRb if and only if $a - b + \sqrt{2}$ is an irrational number. Then the relation R is **2 M**

(A) reflexive only
(B) reflexive and symmetric, but not transitive
(C) equivalence relation
(D) none of these

2 Let $A = \{1, 2, 3\}$. Then the number of relations containing (1,2) and (2,3) which are reflexive and transitive but not symmetric is **2 M**

(A) 1 (B) 2 (C) 3 (D) 4

3 Let $f : [1, \infty) \rightarrow R$, given by $f(x) = 9x^2 - 6x + 5$, then range of f is **2 M**

(A) $[-5, \infty)$ (B) $[-4, \infty)$ (C) $[5, \infty)$ (D) none of these

4 Let R be the set of real numbers. Consider the following functions defined on R. **2 M**

$f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$

$g : R - \{-\frac{4}{3}\} \to R$ defined by $g(x) = \frac{4x}{3x+4}$

Which of the following is true?

(A) f is one-one and onto , g is one-one but not onto

(B) Both f and g are one-one and onto

(C) f is one-one but not onto , g is one-one but not onto

(D) none of these

5 The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is **2 M**

(A) $\sqrt{5} - 2$ (B) $-\sqrt{5} - 2$ (C) $-2 \pm \sqrt{5}$ (D) none of these

6 If $\cos^{-1}x > \sin^{-1}x$, then **2 M**

(A) $\frac{1}{\sqrt{2}} < x \leq 1$

(B) $0 \leq x < \frac{1}{\sqrt{2}}$

(C) $-1 \leq x < \frac{1}{\sqrt{2}}$

(D) $x > 0$

7 If A and B are square matrices of same order, then $AB^T - BA^T$ is a **2 M**

(A) null matrix

(B) unit matrix

(C) symmetric matrix

(D) skew-symmetric matrix

8 If A is a square matrix of order 3 such that $A\,(adj\ A) = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, **2 M**

$B = \begin{bmatrix} 1 & 0 & 0 \\ 7 & -1 & 0 \\ 5 & 2 & -2 \end{bmatrix}$ then the value of $|-2AB|$ is

(A) 12

(B) -1296

(C) 48

(D) -324

9 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ where $A^2 - xA = I_2$ and y, z are the values for which the matrices $\begin{bmatrix} 3y+7 & 5 \\ z+1 & 2-3y \end{bmatrix}$, $\begin{bmatrix} 0 & z-2 \\ 8 & 4 \end{bmatrix}$ are equal, then find $x + y + z$ **2 M**

Ⓐ $\frac{26}{7}$

Ⓑ $\frac{31}{3}$

Ⓒ 3

Ⓓ Not possible to find

10 Find the values of a, b, c for which the function **2 M**

$$f(x) = \begin{cases} \dfrac{\sin (a+1)x + \sin x}{x}, & \text{for } x < 0 \\ c, & \text{for } x = 0 \\ \dfrac{(x + bx^2)^{1/2} - x^{1/2}}{bx^{3/2}}, & \text{for } x > 0 \end{cases}$$

is continuous at $x = 0$

Ⓐ $a = \frac{3}{2},\ b \in R,\ c = \frac{1}{2}$

Ⓑ $a = -\frac{3}{2},\ b \in R,\ c = \frac{1}{2}$

Ⓒ $a = \frac{3}{2},\ b \in R,\ c = -\frac{1}{2}$

Ⓓ none of these

11 If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to **2 M**

Ⓐ $3t$

Ⓑ $\frac{3}{4t}$

Ⓒ $\frac{3}{2t}$

Ⓓ $\frac{3}{4}$

12 If $y = sin^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right)$ then $\frac{dy}{dx} =$ **2 M**

Ⓐ $\frac{1}{2(1+x^2)}$

Ⓑ $-\frac{1}{2\sqrt{1-x^2}}$

Ⓒ $\frac{1}{2\sqrt{1-x^2}}$

Ⓓ $-\frac{1}{2(1+x^2)}$

13 The equation of tangent to the curve $y\left(1+x^2\right) = 2 - x$, where it crosses the $x-$ axis, is **2 M**

Ⓐ $5x + y = 2$

Ⓑ $5x - y = 2$

Ⓒ $x - 5y = 2$

Ⓓ $x + 5y = 2$

14 If the curves $ay + x^2 = 7$ and $y = x^3$ cut orthogonally at $(1, 1)$, then the value of a is **2 M**

Ⓐ 0

Ⓑ -1

Ⓒ 6

Ⓓ -6

15 The function $f(x) = 4sin^3x - 6sin^2x + 12sin\,x + 100$ is strictly **2 M**

Ⓐ increasing in $\left(\pi , \frac{3\pi}{2}\right)$

Ⓑ decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Ⓒ decreasing in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Ⓓ decreasing in $\left(0, \frac{\pi}{2}\right)$

16 $f(x) = (x+1)^3 (x-3)^3$ **2 M**

Ⓐ strictly increasing in the interval $(1, \infty)$

Ⓑ strictly decreasing in the interval $(1, \infty)$

Ⓒ strictly increasing in the interval $(-\infty, 1)$

Ⓓ none of these

17 When x is real, then the minimum value of $x^2 - 8x + 17$ is **2 M**

Ⓐ -1

Ⓑ 0

Ⓒ 1

Ⓓ 2

18 The maximum value of $\left(\frac{1}{x}\right)^x$ is **2 M**

Ⓐ e

Ⓑ e^e

Ⓒ $e^{1/e}$

Ⓓ $\left(\frac{1}{e}\right)^{1/e}$

19 Corner points of the feasible region for an LPP are $(0,2)$, $(3,0)$, $(6,0)$, $(6,8)$ and $(0,5)$. Let $F = 4x + 6y$ be the objective function. The minimum value of F occurs at **2 M**

Ⓐ $(0,2)$ only

Ⓑ $(3,0)$ only

Ⓒ the mid point of the line segment joining the points $(0,2)$ and $(3,0)$ only

Ⓓ any point on the line segment joining the points $(0,2)$ and $(3,0)$

20 The corner points of the feasible region determined by the system of linear constraints are $(0,0)$, $(0,40)$, $(20,40)$, $(60,20)$, $(60,0)$. The objective function is $Z = 4x + 3y$. Compare the quantity in Column A and Column B. **2 M**

Column A	Column B
Maximum of Z	325

(A) The quantity in column A is greater
(B) The quantity in column B is greater
(C) The two quantities are equal
(D) The relationship cannot be determined on the basis of the information supplied.

Section II **16 Marks**

Questions in this section carry 2 marks each.
Both the Case study based questions are compulsory. Attempt all 4 sub-parts of each question

21 A diet is to contain 30 units of vitamin A, 40 units of vitamin B and 20 units of vitamin C. Three types of foods F_1, F_2 and F_3 are available. One unit of Food F_1 contains 3 units of vitamin A, 2 units of vitamin B and 1 unit of vitamin C. One unit of food F_2 contains 1 unit of vitamin A, 2 units of vitamin B and 1 unit of vitamin C. One unit of food F_3 contains 5 units of vitamin A, 3 units of vitamin B and 2 units of vitamin C. Based on the above information, answer the following questions:

a If the diet contains x units of Food F_1, y units of food F_2 and z units of food F_3. What is the matrix equation representing the above situation **2 M**

(A) $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 1 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \\ 20 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \\ 20 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \\ 20 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \\ 20 \end{bmatrix}$

b If A is the coefficient matrix in the above situation, then what is the value of $|adj\ A|$? **2 M**

(A) 2 (B) 4 (C) 8 (D) 16

c What is the value of $\left|A^{-1}\right|$? **2 M**

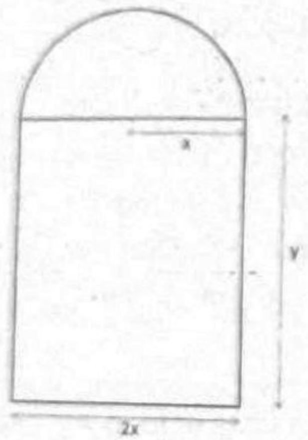
(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{16}$

d What is $A.(adj\,A)$? **2 M**

(A) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

22 Mr. Sharma is an architect.
He designed a building and provided an entry door in the shape of a rectangle surmounted by a semicircular opening.
The perimeter of the door is 10m.



Based on the above information, answer the following questions:

a If $2x$ metres and y metres be the breadth and the height of the rectangular part of the door, then the relation between x and y is **2 M**

(A) $y = 5 - \frac{1}{2}(\pi + 2)\,x$ (B) $y = 10 - \frac{1}{2}(\pi + 2)\,x$

(C) $y = 5 - x - \pi x$ (D) $y = 10 - (\pi + 2)\,x$

b To allow maximum airflow inside the building, the width of the door is **2 M**

(A) $\frac{10}{4+\pi}m$ (B) $\frac{20}{4+\pi}m$ (C) $\frac{20}{2+\pi}m$ (D) $\frac{40}{2+\pi}m$

c To allow maximum airflow inside the building, the height of the door is **2 M**

(A) $\frac{5}{4+\pi}m$ (B) $\frac{10}{4+\pi}m$ (C) $\frac{20}{4+\pi}m$ (D) $\frac{30}{4+\pi}m$

d The area of the door which permits the maximum airflow inside the building is **2 M**

(A) $\frac{100}{4+\pi}m^2$ (B) $\frac{200}{4+\pi}m^2$ (C) $\frac{80}{4+\pi}m^2$ (D) $\frac{50}{4+\pi}m^2$

Part B

24 Marks

Section III **9 Marks**

Questions in this section carry 3 marks each.

23 If R and S are two equivalence relations in a set A. Check whether **3 M**

(i) $R \cap S$ is transitive ....($1\frac{1}{2}$ marks)

(ii) $R \cup S$ is transitive...........($1\frac{1}{2}$ marks)

OR

Show that the relation R defined on the set A of all triangles in a plane $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ is an equivalence relation ....(3 marks)

24 **3 M**

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$

25 Solve the linear programming problem graphically: **3 M**

Minimise $Z = 200x + 500y$

Subject to the constraints:

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

Section IV **15 Marks**

Questions in this section carry 5 marks each.

26 **5 M**

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use this product to solve the system of equations

$x - y + z = 4, x - 2y - 2z = 9 \text{ and } 2x + y + 3z = 1$.

OR

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find out A^{-1} and hence, solve the system of equations

$2x + y - 3z = 13, 3x + 2y + z = 4 \text{ and } x + 2y - z = 8$.

27 **5 M**

If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

28 (i) Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$ **5 M**

(ii) For what value of m is the line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$?